

**Rotational motion** – the motion of an object that is spinning

1. The object spins about an axis.
2. The **axis of rotation** is the line about which the rotation occurs.
3. **Circular motion** is defined as a point on an object that moves (rotates) around an axis.

Since the direction of the motion is constantly changing, linear quantities cannot be used. The circular motion will be **described by the angle through which the object moves**. All points are rotating except the point on the axis.

**Example from the textbook:**

The light bulb on the Ferris wheel is moving about an axis. The axis is a fixed point in the center of the Ferris wheel. Establish a reference line. Use  $0^\circ$  on the right side of a horizontal line. The light bulb is located at a distance  $r$  from the axis as it moves counter clockwise from  $0^\circ$ . It moves through an arc of length  $s$  and through angle  $q$  and through time interval  $dt$ .

Degrees is always used to indicate direction. In science angles are measured in **radians (rad)** rather than degrees. Radians is a pure number with no dimensions and radians is

defined by the following equation:

$$q_{(rad)} = \frac{s}{r}$$

- $q$  - the angle through with the object moves
- $s$  – arc length
- $r$  – length of the radius

Note:  $q$  is a **ratio**. When  $s$  is divided by  $r$ , the **units for length cancel**.

One rotation through a circle is  $360^\circ$ . When an object moves through an angle of  $360^\circ$ , the **arc length** is equal to the **circumference of a circle** -  $2\pi r$ . The arc length of  $360^\circ$  in radians is  $2\pi$  rad (radians) or **6.2831** rad

The equation to convert degrees to radians:

$$q_{(rad)} = \frac{\pi}{180^\circ} \cdot q_{(deg)}$$

If degrees from radians is required, then:

$$q_{(deg)} = \frac{180^\circ}{\pi} \cdot q_{(rad)}$$

The unit for angular motion is radians. Because there are  $2\pi$  radians in a full circle, radians are expressed and multiple of  $\pi$ .

**Angular displacement** – The angle that a point or object has rotated about a specific point (axis). The units are radians. Angular displacement is expressed by the formula:

$$\text{Angular displacement (radians)} = \frac{\text{change in arc length}}{\text{distance from axis}} \quad \text{or} \quad \Delta\theta = \frac{\Delta s}{r}$$

**Angular speed** – The rate at which a point or object rotates about an axis, the time. The units are radians per second (**rad/s** or **w<sub>avg</sub>**). **w** is the small Greek letter for omega and is the symbol for radians per second.

Speed is a scalar quantity of distance (**Dx**) divided by time (**Dt**). Angular speed is angular displacement (**Dq** in rads) divided by time (**Dt**). The formula is:

$$\omega_{\text{avg}} = \frac{\Delta q(\text{rads})}{\Delta t}$$

**Angular acceleration** – The rate of change of angular speed, an object's angular speed either increases or decreases as it moves through the arc. The units are radians per second per second (**rad/s<sup>2</sup>**). The small Greek letter alpha (**a**) is used for angular acceleration. The formula is:

$$\alpha_{\text{avg}} = \frac{w_2 - w_1}{t_2 - t_1} = \frac{\Delta w}{\Delta t}$$

### Comparing Angular and Linear Quantities

The equations are all similar. **X** (linear distance) is replaced with **Dq** (radians), **V** (m/s) is replaced with **w** (Dq /s), and **a** (m/s<sup>2</sup>) is replaced with **a** (Dq /s<sup>2</sup>). Below is a table that compares rotational to linear kinematics equations. Note the similarities and the differences between these two sets of equations.

Rotational motion with constant angular acceleration	Linear motion with constant acceleration
$w_{\text{avg}} = \Delta q / \Delta t$	$V_{\text{avg}} = \Delta x / \Delta t$
$w_f = w_i + a \Delta t$	$v_f = v_i + a \Delta t$
$\Delta q = w_i \Delta t + 1/2 a (\Delta t)^2$	$\Delta x = v_i \Delta t + 1/2 a (\Delta t)^2$
$w_f^2 = w_i^2 + 2 a (\Delta q)$	$v_f^2 = v_i^2 + 2 a (\Delta x)$
$\Delta q = 1/2 (w_i + w_f) \Delta t$	$\Delta x = 1/2 (v_i + v_f) \Delta t$

**Tangential speed** – the instantaneous linear speed of a point rotating about an axis at that point

Remember a **tangent** is a line that touches one point on a circle and the line is perpendicular to a line from that point to the axis.

Any point on a rotating wheel will have the same angular velocity but they will have different tangential speeds. Why? The amount of radians at different radii from the axis of the wheel will be the same for a given time but the tangential speed will increase proportional with the radius of the wheel.

**To calculate tangential speed:**

A bicycle wheel rotates through an angle  $\Delta\theta$  and a nail in the tire rotates through an arc length  $\Delta s$  for interval  $\Delta t$ . What is the tangential speed?

1. Solve for angular displacement. 
$$\Delta\theta = \frac{\Delta s}{r}$$

2. Solve for the tangential speed of the nail. Divide both sides of the equation by the time the nail takes to travel distance  $\Delta s$ .

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r} \frac{\Delta s}{\Delta t}$$

3. Tangential speed is equal to the distance from the axis times the angular speed  
or  
$$v_t = r\omega$$

Note:  $\omega$  is instantaneous angular speed. This equation is valid **only** when  $\omega$  is measured in radians per unit of time.

**Tangential Acceleration** – instantaneous linear acceleration is tangent to the circular path

The bicycle wheel speeds up the nail will have angular acceleration. The linear acceleration related to this angular acceleration is tangent to the circular path. This **instantaneous linear acceleration** is called the **tangential acceleration** and is expressed with the following formula:

Tangential acceleration = distance from the axis  $\times$  angular acceleration

or

$$a_t = r\alpha$$

**Centripetal acceleration** – the acceleration of an object toward the center of a circular path

Linear acceleration magnitude is expressed by the formula:

$$a = \frac{V_f - V_i}{t_f - t_i} \quad \text{or} \quad a = \frac{\Delta v}{\Delta t}$$

Acceleration depends on a change in velocity and or direction. An object moving in a circular path at a constant speed is accelerating because it has a constant change in direction. If you are riding in a car that is moving at a constant speed in a circle, you will feel the acceleration because your body will feel a **force** pushing you seemingly outward (more on this later) from the center of the circle. The formula for **centripetal acceleration** magnitude is:

$$\text{Centripetal acceleration} = (\text{tangential speed})^2 / \text{distance from axis}$$

or

$$a_c = \frac{V_t^2}{r}$$

and since tangential speed is related to the angular speed, another formula for finding centripetal acceleration magnitude is:

$$\text{Centripetal acceleration} = \text{distance from axis } \mathbf{X} (\text{angular speed})^2$$

or

$$a_c = r\omega^2$$

Tangential and centripetal accelerations are perpendicular to each other. Because the object is moving in a circular path, it always has centripetal acceleration due to the constant changing of direction. If the speed is changing, then the object also has tangential acceleration. Because these components are perpendicular to each other, Pythagorean's theorem can be used to solve for the **magnitude** of the *total acceleration* of the system.

$$a_{total} = \sqrt{a_t^2 + a_c^2}$$

The direction of the total acceleration of the system can be found by using the inverse of the tangent function.

$$\theta = \tan^{-1} \frac{a_c}{a_t}$$

Note that:

1. An object that moves at a **constant speed in a circular motion** has
2. a velocity vector that **is continuously changing direction** has
3. **centripetal acceleration** directed toward the center of the motion.

The magnitude for the object is determined by:  $a_c = \frac{V_t^2}{r}$

If the object is attached to a string and assuming a constant speed, the string exerts a force that counteracts the tendency of the object to maintain a straight-line path. The magnitude of the straight-line force can be calculated by  $F_c = ma_c$

The force that maintains the circular motion is directed toward the center of the object's circular path, the force exerted by the string. Two formulas that calculate the force are:

1. Force that maintains circular motion = mass X  $\frac{(\text{tan gential speed})^2}{\text{dis tan ce to axis}}$

$$\text{or } F_c = \frac{mv_t^2}{r}$$

2. Force that maintain circular motion = mass x distance to axis x (angular speed)<sup>2</sup>

$$\text{or } F_c = mr\omega^2$$

This force (Newton) is not different from any other force. A **force** is the **push or pull** on an object that **cause a change in velocity and/or direction**.

The **force** that maintains circular motion **acts at right angles** to the motion. It **causes a change in the direction** of the velocity. Example: the string attached to the object.

If the force vanishes, then the object will follow a straight-line path.

### The motion of a rotating system

You are a passenger in a car on a left curved ramp traveling at a high speed. You slide right to door. The force of the door keeps you from sliding out of the car. What is the force that pushes you outward? **None**. You have a tendency to travel in a straight-line path and **inertia** is that tendency to keep traveling in a straight-line path. Which of Newton's laws is this? \_\_\_\_\_ Remember that **inertia is not a force**.

## Newton's Law of Universal Gravitation

Why do our planets stay in the sun's orbit?

Why does the moon stay in orbit around the Earth?

Why can an object not ever be completely weightless?

**Gravitational force (field force)** is the **mutual force of attraction between particles** of matter. It is a field force that always exists between any two masses, regardless of the medium that separates them. It is this gravitational force keeps the planets from drifting out of orbit.

The gravitational force is **directly proportional to the product of the masses** between the objects and the **distance between the centers of the masses**. The force can be very small (a penny to Earth) or very large (Jupiter to the Sun). This also applies to the gravitational force between the Earth and the moon that will be used as an example.

Newton's 3<sup>rd</sup> law of motion *applies* to gravitational force because the force exerted on Earth by the moon ( $F_{mE}$ ) is equal in magnitude and in the opposite direction of the force exerted on the moon by Earth ( $F_{Em}$ );  $F_{Em} = F_{mE}$ .

The equation for Newton's law of universal gravitation is:

$$\text{Gravitational force} = \text{constant} \times \frac{\text{mass1} \times \text{mass2}}{(\text{distance between center of masses})^2}$$

$$\text{or } F_g = G \frac{m_1 m_2}{r^2}$$

**G** is the constant of universal gravitation that has been determined experimentally. It is used to calculate the gravitational forces between any two particles.

$$G = 6.673 \times 10^{-11} \cdot \frac{N \cdot m^2}{kg^2}$$

Note that the **universal gravitation** is an example of an **inverse-square law** because the **force** between two masses **decreases as the masses move farther apart**.

Earth and moon is used as an example. The mass of Earth is  $5.98 \times 10^{24}$  kg and the mass of the moon is  $7.35 \times 10^{22}$  kg. Since the two particles orbit around each other, the distance is the radius and the radius is  $3.844 \times 10^5$  meters.

$$F_g = G \frac{m_1 m_2}{r^2}; \quad F_g = 6.673 \times 10^{-11} \cdot \frac{5.98 \times 10^{24} \times 7.35 \times 10^{22}}{(3.844 \times 10^5)^2}; \quad F_g = 1.9 \times 10^{26} \text{ N}$$