Introduction to Vectors

A **vector** is a physical quantity that has **both** magnitude and direction.

An example is a plane flying NE at 200 km/hr. This vector is written as **200 Km/hr at 45°**. Another example is Fred is pushing a table with a force of 125 Newtons to the left or **125 Newtons at 180°**. A vector quantity is always written with **boldface** type.

A **scalar** is a physical quantity that has magnitude but no direction.

Look at the speedometer of the car. The speedometer indicates the speed of the car at that instant and it **does not** show direction. Other examples of scalar quantities are the speed sound, speed of light, the number of pages in a book or the number of students in a classroom. Scalar quantities are always written with *italics*.

The speed of a car written in both scalar or vector format:

scalar: v = 50 km/hr

vector: **v** = 50 km/hr northwest or $\vec{v}$ = 50 km/hr northwest

In the above example, the italics "v" with an arrow above it "$\vec{v}$" is used for ***writing*** the vector.

In the case of multiple vectors, **draw a diagram**. Example, a plane is flying East at 900 km/hr and a wind of 100 km/hr is blowing south. Vectors on paper can be drawn to indicate the magnitude (length of vector) and direction of the plane and wind.

Vectors can be combined only if they have the same units. Feet and meters cannot be combined. Do dimensional analysis to convert the feet to meters and then combine the meters and meters.

Graphing vectors:

1. The vectors must have the same units.
2. Same direction and opposite direction vectors can be combined by adding.
3. Draw the vectors to scale on graph paper.
4. Total displacement (magnitude) can be found by measuring with a rule.
5. The direction can be determined with a protractor.

The singular remaining vector left after the combining process is the **resultant**.

resultant – a singular vector that results in the sum of two or more vectors

Vector Properties

1. When two or more vectors act at the same point the resultant vector will have the same effect as the combined individual vectors.
2. Vectors that have the same units, same direction, and different magnitudes can be combined by addition.
3. Vectors that have the same units, opposite directions, and different magnitudes can be combined by subtraction.

Vector Operations

Coordinate systems in Two Dimensions – When the units are the same and when the direction of the vectors are the same or opposite, combining is easy through addition or subtraction. If the direction of the vectors is different, a coordinate (graph) system must be used to combine the vectors.

The coordinate system convention:

1. *Y*-axis is to the North and South. *X*-axis is to the East and West.
2. *Y*-axis is vertical (perpendicular to the ground) and the x-axis is horizontal.
3. **Be consistent with the coordinate system.**

Note:

1. There are no firm rules for applying coordinate systems to situations involving vectors.
2. Be consistent, the final answer will be correct regardless of the coordinate system used.
3. The best choice for orienting the axis is the approach that makes solving the problem the easiest.

The universal convention for direction of travel is North is 0°, East is 90°, South is 180° and West is 270°.

Solving Magnitude and Direction of the resultant vector – Two parts:

1. **Pythagorean Theorem** – Use Pythagorean's theorem when the magnitudes are known and the vectors are perpendicular.

$$c^2 = a^2 + b^2$$

$$c(\text{length of hypotenuse})^2 = a(\text{length of one side})^2 + b(\text{length of other side})^2$$

When using Pythagorean's theorem decide which side will be Δx and which side will be Δy. The opposite side displacement will be the magnitude.

2. **Tangent Function** – Use the tangent function to find the direction of the resultant vector when the vectors form a right triangle.

Tangent function: $\tan\theta = \frac{opp}{adj}$

Inverse tangent function: $\theta = \tan^{-1}\left(\frac{opp}{adj}\right)$

Use the inverse tangent function to obtain the angle/direction of the resultant vector. Decide which side will be Δx and which side will be Δy.

Note: An accurate protractor and rule is useful for making very close measurements of the vectors to check your math.
Note: Practice estimating your answers with out doing any math or measuring.

Resolving Vectors into Components

Vector Components:
1. horizontal parts
2. vertical parts
3. displacement
4. angles between the parts

x component is parallel to the *x*-axis.
y component is parallel to the *y*-axis.
Components can have either positive or negative numbers.

Each vector has **two components**: 1. magnitude 2. direction

By resolving the vector the motion is more conveniently described in terms of speed and direction. An example is 95 km/hr north/northeast. More specifically would me 95 km/hr at 20°.

Resolving the Magnitude and Direction of the resultant vector – Two parts:

Assume the givens are the hypotenuse and the angle of the hypotenuse. While using the below formulas, determine which side will be represented by Dx and which side will be represented by Dy. Rearrange the formulas to solve for the respective vectors.

1. Sine function – solves for one side of a right triangle:

$$\sin q = \frac{opp}{hyp} \quad \rightarrow \quad \sin q = \frac{v_y}{v} \quad \rightarrow \quad v_y = v(\sin q)$$

2. Cosine function – solves for another side of a right triangle

$$\cos q = \frac{adj}{hyp} \quad \rightarrow \quad \cos q = \frac{v_x}{v} \quad \rightarrow \quad v_x = v(\cos q)$$

Adding Vectors that are not Perpendicular

The direction and magnitude of a moving object may be constantly changing. Can the Pythagorean theorem be applied? Not yet. The movement of the object is broken down into various straight-line components. Each straight-line component can be resolved into the Dx and Dy components and the respective Dx and Dy components are added together. Pythagorean's theorem is used to solve for the final resultant vector and the tangent function is used to find the direction.

Example problems and how to solve:

1. Determine the magnitude and direction of the vector in figure 1. Two methods can be used:

- **a.** Graphically
- **b.** Mathematically

The graphical method involves a scale and a protractor. It is very quick to obtain accurate measurements of magnitude and direction.

The mathematical process requires Pythagorean's Theorem and trig functions. It takes longer but the answers are more precise for possible future mathematical computations.

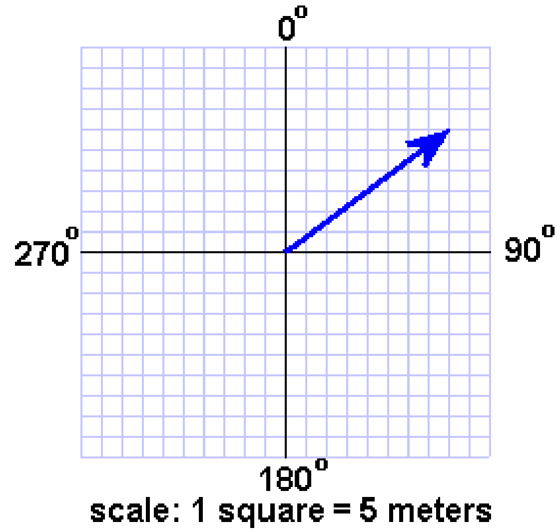


Figure 1

First, in figure 2, draw in a right triangle, the arrow is the magnitude and also is the hypotenuse of the right triangle. Next label the sides of the triangle and last label the angle that shows the quantity of the direction.

First calculate the magnitude using Pythagorean's Theorem $a^2 + b^2 = c^2$. From the figure, a = 30m and b = 40m. Calculations determine that **c = 50m** for the magnitude of the arrow.

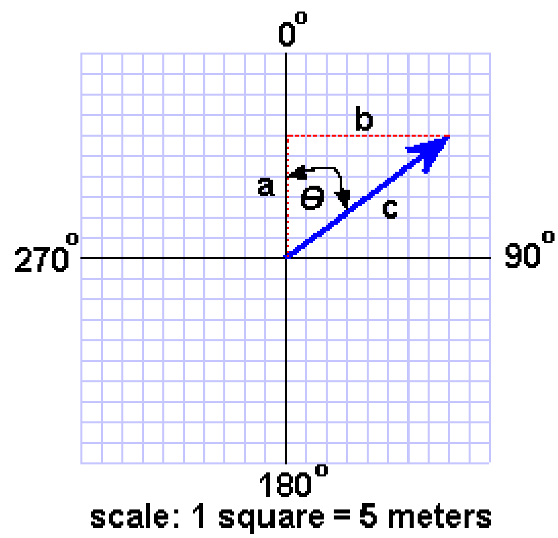


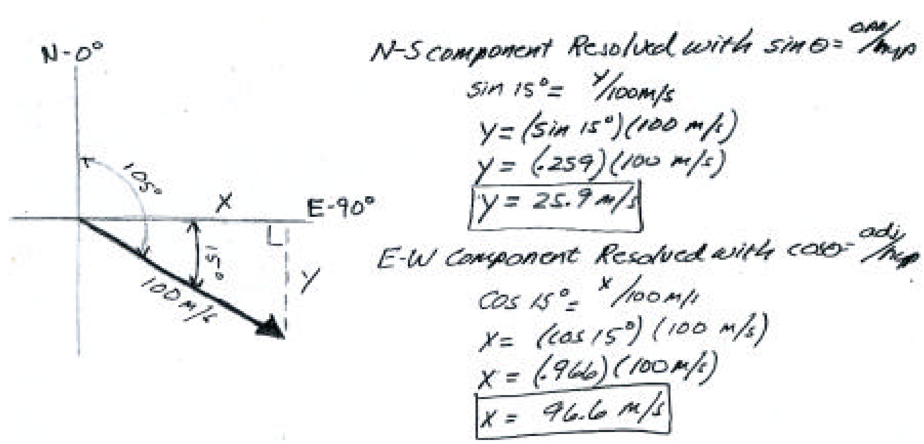
Figure 2

Second calculate the direction of the magnitude. Notice that 0° is at the top of the graph so this coordinate system is geographically based on North (0°), East (90°), South (180°), and West (270°). Any of the trig functions can be used and the answers will be the same.
tan$q$ = opp/adj has been chosen this time. The function is algebraically rearrange to solve for q.
$q = \tan^{-1}(b/a) = \tan^{-1}(40/30) =$ **53.1°**.

2. Resolve a vector of **100 m/s** operating at an angle of **105°**. Precisely graph the vector.

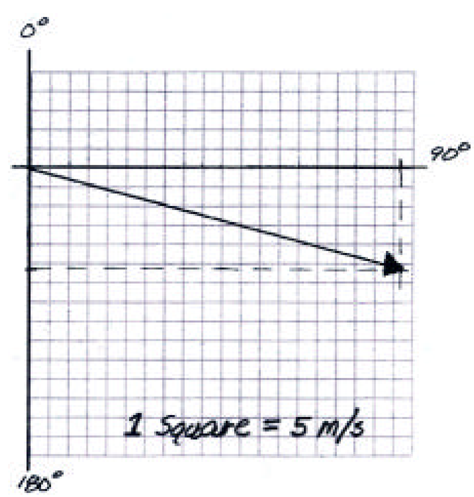
When vectors are combined, the raw angles and magnitudes values cannot be added or subtracted from each other. The North-South (y-axis) quantities can be combined and the East-West (x-axis) quantities can be combined. Therefore the respective vectors of an event must be resolved into their individual components that includes the magnitude of the vector along the N-S axis and the E-W axis.

First, make a quick sketch of the event as shown below. The sketch is not intended to be accurate or precise but used as a guide to resolve the vector. **100 m/s** is the magnitude and will also be the hypotenuse of the right triangle. **105°** is measured from North or **0°** and is **15°** below the East-West axis.



Note: When solving for the y-axis value, this component is extending in the negative direction along the y-axis, therefore it is a negative number for the direction and for combining with other vector "y" values.

To the right is a precisely drawn vector that shows the magnitude and direction. Always include a scale. The tip of the vector is at (96.6 m/s, -25.9 m/s) and the angle is 105° or E 15° S.



3. In the figures below solve for the resultant vector.

This process involves finding one vector that replaces two or more vectors by combining vectors. This process can be done graphically provided the correct equipment is available to produce accurate results or mathematically to produce precise results. Resolve each vector into it's respective **x** and **y** coordinate. Combine the **x** and **y** coordinates, use Pythagorean's Theorem to find the magnitude then use a trig function to determine the direction.

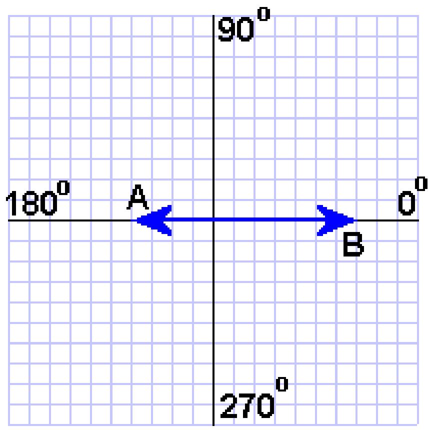


Figure 1

scale: 1 square = 4 N

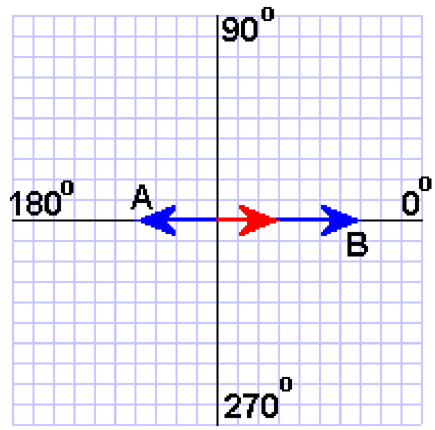


Figure 2

Think of two groups of people playing tug-o-war. The problem in figure 1 fits that scenario perfect. The vector **A** has a magnitude of **–16 N** at **180**° and the vector **B** has a magnitude of **28 N** at **0**°. Since the vectors are straight-line vectors, they are combined by: - 16 N + 28 N = **12 N** at **0**°. The solution is the red vector shown in figure 2.

4. In the problem below, solve for the resultant vector.

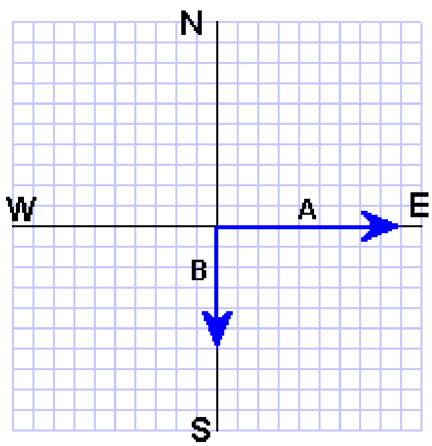


Figure 3

scale: 1 square = 0.5 m/s

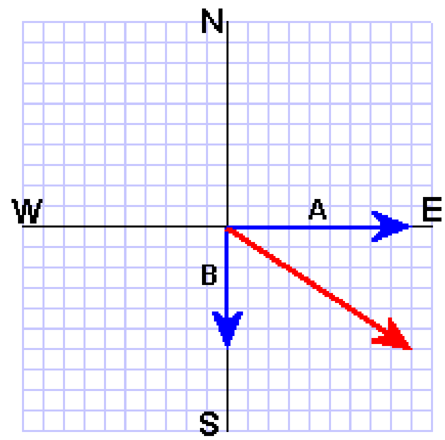


Figure 4

Think of problem in figure 3 as a boat moving across a river. Vector **A** is the velocity of a boat in still water. Vector **B** is the velocity of the water flow in a river. Solve for the velocity of the boat. Figure 4 is the solution and shows the direction of the boat.

Vector A is resolved into 4.5 m/s at 90° and 0 m/s at 180°.
Vector B is resolved into 0 m/s at 90° and 3 m/s at 180°.

Combine the E-W vectors: 4.5 m/s + 0 m/s = 4.5 m/s East (x).
Combine the N-S vectors: 0 m/s - 3 m/s = - 3 m/s South (y).

Solve for the magnitude of the resultant (the direction the boat will travel) with Pythagorean's Theorem: 4.5 m/s^2 + 3 m/s^2 = 29.25 m/s^2. **Magnitude = 5.4 m/s**

Solve for the direction of the resultant vector.
Tan*q* = opp/adj; *q* = tan^{-1}(3 m/s **/** 4.5 m/s) *q* = 33.7°
Direction = E 33.7° S or **213.7°**